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Mistakes Made by Students While Posing Problems for Equations Containing Algebraic Fractional Expressions*

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Abstract: Conceptual and procedural knowledge has historically been one of the most discussed subjects in mathematics. In this study, errors in equations including algebraic fractional expressions were examined to determine where ninth-grade students in their comprehension of these expressions. The study employed a holistic multiple-case strategy, and the gathered data were evaluated using content analysis. The results indicate that students made the most mistakes with the definitions of quotient and fraction-whole. Certain sorts of errors in the job are applicable to all fraction interpretations, while others appear to be fraction-specific. Depending on the type of fraction, the mistake circumstances varies.

Keywords: Algebraic fractions, verbal problems, problem posing, mistakes

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Introduction

Numbers and operations are an integral part of mathematics. However, mathematics is much more than that. To develop meaning about mathematical concepts understand, it is necessary to have the conceptual knowledge (Hiebert & Carpenter, 1992, Star, 2001).

Many definitions of conceptual knowledge (Brownell, 1935; Byrnes & Wasik, 2009; Garofalo & Lester, 1985; Greeno, 1978) have been made in the past. Crooks and Alibali (2014) based on an intensive literature review, existing definitions of conceptual knowledge are related knowledge (relationships within a domain; Hiebert & Lefevre, 1986), knowledge of general principles (general rules, facts, and definitions; De Jong & Ferguson-Hessler, 1996), knowledge of principles on which procedures are based (foundations of principles; Pardhan & Mohammad, 2005), category knowledge (categories that organize knowledge; Byrnes, 1992), domain structure knowledge (organization of mathematics; Robinson & Dube, 2009), and symbol knowledge (symbol meanings; Ploger & Hecht, 2009) divided into six categories.

This study focused on students' symbol knowledge in the context of equations containing algebraic fractional expressions. Alibali et al. (2014) stated that symbols in equations are important in conceptual understanding and that the formation of algebraic thinking depends on understanding symbols.

Chae (2005) stated that symbolic representation could be transformed into verbal story representation, tabular representation, or graphic representation. Instead of giving symbols alone without any context, giving them concerning these references (e.g., diagram, table, verbal explanation) is important for learning equations in a meaningful way (Alibali et al., 2014; Kopparlaa et al., 2019; Panasuk & Beyranevand, 2010).

As these references are helpful in the interpretation of symbols, the operations in which symbols are used together are also important in terms of the meanings attributed to symbols. Algebraic fractions are symbolic representations that can be formed differently ways, used in different operations, and therefore have different meanings. For example, x/y can represent the ratio of the numbers x and y, and can also be represented as x over y. The meanings ascribed to algebraic fractions can be affected by different representations and meanings of fractions.

Fractions can be represented in four different ways: verbally, symbolically, as an object, and as a model (Birgin & Gurbuz, 2009). While verbal representation is the expression of fractions in spoken language (two out of seven, etc.), symbolic representation is the representation with numbers (like 1/2) or symbols (like a/b). Object representations are representations made using concrete objects. Model representations are conceptual systems developed to mathematically describe, explain, interpret, and represent a situation (Lesh & Doerr, 2003). Model representations used in fractions are length, area, and set. It is important to use different representations of mathematical concepts and to make transitions between these representations in terms of providing conceptual understanding (Bossé, Adu-Gyamfi & Cheetham, 2011)

Algebraic fractional equations are thought to have a more complex structure because both algebraic symbols and fractions have different representations. One of the things that complicate the fraction is the different meanings it has (quotient meaning, measure meaning, ratio meaning, operator meaning, part-whole meaning). The meaning of division comes from the logic of equal sharing (Empson, 1995). Measuring means identifying a length and then using that length to measure another object's length (Van De Walle, Karp and Bay-Williams, 2012). Ratio means the ratio of two quantities to each other (Acar, 2010). It is the case when the rational number in the operator sense is the main element of an operation. For example, if you are asked to find 1/3 of the flowers in a vase, the rational number 1/3 here acts as an operator. Ratio meaning involves comparing two quantities (Acar, 2010). In the sense of part-whole, students break the given objects and learn to express the resulting parts as “fractions” according to the unit whole (Toluk, 2001).

This research discusses the verbal representation of the symbolic representation of equations with algebraic fractions. In the problems posed by the students about the addition of fractions, there were seven categories expressing the second fraction over the remainder of the whole, not establishing the part-whole relationship, attributing a natural number meaning to the result of the operation, unit confusion, attributing the natural number meaning to the collected fraction numbers, not reflecting the operation to the root of the question, and failing to attribute meaning to the whole parts of integer fractions (Isik & Kar, 2012).

Similar situations were encountered in studies conducted with pre-service teachers (Basturk, 2016; Akcay & Ardic, 2020). In addition, teachers have difficulty teaching fractions (An, Kulm & Wu, 2004; Izsak, 2008). The same is true for algebraic expressions. For example Isik and Kar (2012) examined the problems posed by pre-service teachers about equations with one and two unknowns of the first degree and they found that pre-service teachers made mistakes such as incorrectly translating mathematical notations, assigning unrealistic values to unknowns, and posing problems by changing the structures of equations.

In this research, the verbal representation of the symbolic representation of equations with algebraic fractions is discussed. In order to determine the mistakes made in the conceptual knowledge dimension in equations with algebraic fractions, problem posing skill, which is a dimension of problem solving skill, was examined. Because most of the time, the individual uses conceptual knowledge in this process (Roth, Jones & Idol, 1990).

Problem posing was added to Polya's problem-solving methodology as the fifth step by Gonzales (1994). Problem posing is an important part of research and practice in school mathematics and is regarded as a critical intellectual activity in scientific research for a long time (Pirie, 2002; Cai, Hwang, Jiang & Silber, 2015). Silver (1994) defined problem posing as either the creation of new problems or questions or the reframing of a given problem to investigate a given situation.Problem posing supports students' conceptual understanding and helps teachers to see students' deficiencies in that concept (Ayllón, 2005).

Some methods for problem posing include formulating a problem under some given conditions (a representation, a required context, a specific operation) (Christou, etc., 2005; Stoyanova, 1998). The semi-structured problem-posing situation was used in this study. In this context, the mistakes made by students while posing verbal problems related to daily life about equations with algebraic fractions were examined.

In the literature, there are many studies on the fractions such as (Bunar, 2011; Kavuncu & Yenilmez, 2021), operations in fractions (Barlow & Cates, 2006; Isik & Kar, 2012; Koichu, Harel & Manaster, 2013; Aydogdu Iskenderoglu, 2018; Martinez & Blanco, 2021) and algebraic expressions (Stephens, 2003; Isik & Kar, 2012; Alibali et al., 2014) to create a verbal problem and mistakes made in this process. Although there are very few studies on posing problems with equations with algebraic fractions (Tastepe & Yanik, 2021), there has been no study on the mistakes made in this process. In this direction, besides helping to eliminate the deficiencies in this context, it will also contribute to student education.

To be successful in problem posing, students always ask themselves questions like "What… changed?", "What if…?" and "What if ... not?" when they face a math problem, problem situation, or the answer to a problem (Ghasempour, Bakar & Jahanshahloo, 2013). They also resort to several strategies (Ghasempour, Bakar & Jahanshahloo, 2013) such as "What if" or "What if not" strategy (Brown & Walter, 2005), imitation strategy (Kojima, Miwa & Matsui, 2009), effective questioning strategy (English, 1997).

Problem posing has been an important part of research and practice in school mathematics and is regarded as a critical intellectual activity in scientific research for a long time (Cai, Hwang, Jiang & Silber, 2015). This study aimed to determine the mistakes made by 9th-grade students while posing problems regarding equations with algebraic fractions.

Methodology

A holistic multiple-case design was used to determine the mistakes made by 9th-grade students while posing problems regarding equations with algebraic fractions. In this design, there is more than one situation; each situation is examined in a holistic way and compared with each other (Yin, 2018). In this research both in the context of different meanings of the fraction (Part meaning, processor meaning, part-whole meaning, ratio meaning, measurement meaning) and in the context of equations involving different algebraic fractions (numerator algebraic denominator numeric (NaDn), numerator numeric denominator algebraic (NnDa) and algebraic numerator and denominator (NaDa)) were discussed. Each case was studied and compared in terms of strategies used in problem posing.

Participants

First of all, to identify low and medium level participants in writing problems to equations containing algebraic expressions, pre-assessment with 6 open-ended questions was implemented to 240 ninth grade students studying in a small city in the northern region of Turkey. First, participants who could write at least 4 problems correctly or incorrectly in the problem-posing test were determined and the criterion sampling method was used. Then the convenience sampling was used and four voluntary ninth grade students with three low scoresith three low scores (writing one problem) and one medium score (writing two) were selected from this test. Table 1 shows the types of equations (numerator algebraic denominator numeric (NaDn), numerator numeric, denominator algebraic (NnDa), numerator and denominator algebraic (NaDa)) in the pre-test and the true-false or incompleteness of the problems written by the students and their frequencies.

Table 1.

Information on completing the problem-posing task of the participants

Kind of Algebraic expression	Question number	Participant 1 (P1)	Participant 2 (P2)	Participant 3 (P3)	Participant 4 (P4)
NaDn	3 question	1True 1Missing 1False	2True - 1False	2True - 1False	1True - 1False
NnDa	2 question	- - -	-	- - 2False	- 1Missing 1False
NaDa	1 question	- - -	- - -	- 1Missing -	- - -
Score level		Low	Medium	Low	Medium

Table 1 shows that the rate of completing the problem test of the participants varied between 16% and 33%. The rate of completed problem posing for the NaDn 66%, NnDa 0%, NaDa 0%. Within this information, the study group made a normal diversity selection.

Data Collection Tools

A problem posing test consisting of 29 questions has the equation different algebraic fractions (13 questions numerator algebraic numeric, 8 questions numerator numeric numeric numeric numeric and 8 questions numerator and numerator algebraic). The problem-posing test was applied in 3 different sessions at one-week intervals, depending on the fraction type. Students were asked to pose verbal problems related to daily life following the data in the algebraic fractional expression given.

In this study, a think-aloud protocol and a semi-structured interview form consisting of eight main questions and various side questions developed by the researchers were used. The think-aloud protocol is when individuals perform a task and verbally express everything that crosses their minds during task performance (Jääskeläinen, 2010).

Data Analysis

The obtained data were analyzed using by content analysis method. "Content analysis is a method for analysing the content of a variety of data, such as visual and verbal data. It enables the reduction of phenomena or events into defined categories to better analyse and interpret them" (Harwood & Garry, 2003, pp. 479).

Similar data were brought together within the framework of certain concepts (meanings of fractions, types of fractions used) and themes (mistakes made) and organized in a way that the reader can understand.

Validity and Reliability of the Study and Ethics

Triangulation, researcher's position, maximum variation, adequate engagement in data collection, rich and thick descriptions and audit trail, which are strategies for promoting validity and reliability (Merriam & Tisdell, 2016) were used in this study. The triangulation was employed by using multiple data collection tools semi-structured interviews, observations, problem posing papers and think-aloud protocol to verify findings.

The maximum variation strategy was used in sample selection. In this study participants consisted of 4 ninth-grade students. They had a different problem-posing levels and the rate of completing the problem test of them varied. One of the researchers spent two hours with each participant while interviewing them their levels for adequate engagement in the data collection strategy.

Regarding ethics, first of all, the necessary permissions were obtained from student, parent and teacher of the student. Also, volunteer students were in the study, the participants' identity information was kept confidential, and codes were given from P1 to P4. Necessary permissions were obtained from the Directorate of National Education to conduct the research. The students participating in the research were informed about the research process and participant rights, and written permission was obtained from the students, teachers and parents of the students. The interviews were held in public places such as libraries and cafes. Interview records and data were not shared with anyone other than the researchers (Yildirim & Simsek, 2013).

Results

In this study, the participants were asked to write problems for different algebraic fractional equations. In Table 2, It is seen that the participants tried to write verbal problems for the given equations (146 problems for 29 equations), but they wrote incorrect problems (f=37). The frequencies and percentages of the errors made in this process are given in Table 2 in the fraction type category.

Table 2.

Errors Made by Fraction Types

Meaning of Fraction	Fraction Type							
	NaDn		NnDa		NaDa		Total	
	f	%		%		%	f	%
Number of questions asked	13	44,82	8	27,58	8	27,58	29	100
Number of problems posted	66	45,20	37	25,34	43	29,45	146	100
Correct	43	65,15	28	75,67	34	79,06	105	71,91
Missing	2	3,03	1	2,70	2	4,65	5	3,42
Incorrect	21	31,81	8	21,62	7	16,27	36	24,65
Empty	0		1		1		2	
Type of error made	1 Dnfe**** 14 CE 2 NQ 4 MV*	4,76 66,66 9,52 19,04	1 Dnfe 3 CE** 3 Cnd*** 1 AMC*****	12,50 37,50 37,50 12,50	1 Dnfe 5 CE 1 MV	14,28 71,42 14,28	3 Dnfe 22 CE 2 NQ****** 5 MV 3 Cnd 1 AMC	8,33 61,11 5,55 13,88 8,33 2,77

MV*: Meaning of the variable, CE**: Changing the equation, Cnd***: Confusing the numerator and denominator, Dnfe****: Don't act like a non-fractional equation, AMC*****: Adding mathematical content, NQ******: No question

According to Table 2, the participants could write 146-word problems into the 29 equation containing algebraic fractional expressions. Although most of the written word problems are correct, some problems incorrect and incomplete problems. In the problems written, it was seen that the most common mistakes were "changing the equation (CE)", while the other mistakes were much less than each other. According to the equations containing different algebraic fractional expression types, the most common mistake was "changing the equation (CE)". The error of "confusing the numerator and denominator (Cnd)" in equations containing algebraic fractional expressions in the numerator numerical denominator algebraic (NnDa) was the other most common error. The least common error was adding mathematical content (AMC). Other common errors are the meaning of the variable (MV), don't act like a non-fractional equation (Dnfe) and no question (NQ). Table 3 shows the errors made according to the meanings and types of fractions.

Table 3.

Errors Made According to the Meanings of Fractions and Their Types

Meaning of Fraction	Fraction Type							
	NaDn		NnDa		NaDa		Total	
	f	%		%		%	f	%
Quotient meaning	47	71,21	13	35,13	7	15,90	67	
Correct	29	61,70	11	84,61	6	85,71	46	68,65
Missing	1	2,12	1	7,69	1	14,28	3	4,47
Incorrect	17	36,17	1	7,69	0	0	18	26,86
Type of error made	1 Dnfe 12 CE 1 NQ 3 MV		1 Cnd		-		1 Dnfe 12 CE 1 NQ 3 MV 1 Cnd	5,55 66,66 5,55 16,66 5,55
Measurement meaning	8	12,12	9	27,02	14	31,81	32	
Correct	6	75	7	77,77	7	50	20	62,50
Missing	0	0	0	0	1	7,14	1	3,12
Incorrect	2	25	2	22,22	6	42,85	10	31,25
Type of error made	1 MV 1 CE		1 CE 1 AMC		5 CE 1 MV		2 MV 7 CE 1 AMC	20 70 10
Ratio meaning	2	3,03	13	35,13	19	43,18	34	
Correct	1	50	8	61,53	18	94,73	27	79,41
Missing	1	50	0	0	0	0	1	2,94
Incorrect	0	0	5	38,46	1	5,55	6	17,64

Type of error made	0	0	1 Dnfe 2 CE 2 Cnd	100	1 Dnfe	100	2 Dnfe 2 CE 2 Cnd	33,33 33,33 33,33
Operator meaning	7	10,60	0	0	0	0	7	
Correct	5	71,42	0	0	0	0	5	71,42
Missing	0	0	0	0	0	0	0	0
Incorrect	2	28,57	0	0	0	0	2	28,57
Type of error made	1 CE 1 NQ	100	0	0	0	0	1 CE 1 NQ	50 50
Part-whole meaning	2	3,03	2	5,40	3	6,81		
Correct	2	100	2	100	3	100	7	100
Missing	0	0	0	0	0	0	0	0
Incorrect	0	0	0	0	0	0	0	0
Type of error made	0	0	0	0	0	0	0	0

MV*: Meaning of the variable, CE**: Changing the equation, Cnd***: Confusing the numerator and denominator, Dnfe****: Don't act like a non-fractional equation, AMC*****: Adding mathematical content, NQ******: No question

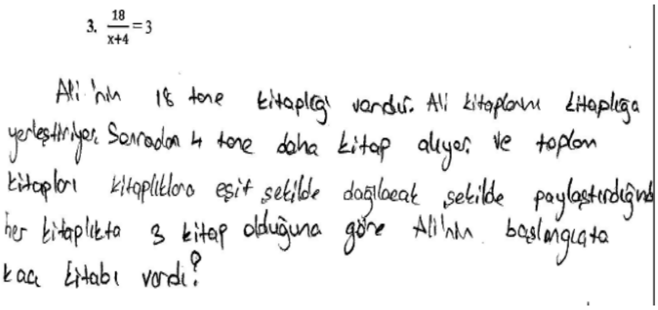
According to Table 3, it was determined that the students made the most mistakes in the sense of quotient meaning, and they did not make any mistakes in the sense of part and whole of the fraction. For this research, the mistakes made by the participants while posing problems with equations with algebraic fractions were examined within the framework of their understanding of fractions (Quotient meaning, measurement meaning, ratio meaning, operator meaning, part-whole meaning).

Quotient Meaning

According to the data obtained, the most errors (50%) were made in fractional quotient. There are five different types of errors regarding this meaning of fraction. It has been seen that the equations with the most errors are those containing algebraic fractional expressions whose numerator is algebraic and whose denominator is numeric. The fraction type that has the most problems with the quotient of the fraction has been algebraic fractional expressions whose numerator is algebraic denominator is number. There are five different types of errors regarding this meaning of fraction. The most common error in the problems written about the quotient of this fraction type was changing the equation (CE). This type of error is one of the most common errors in other fractions' meanings. In Figure 1, there are examples of errors made in this sense of the fraction.

Figure 1.

Verbal Problem Example for Confusing the Numerator and Denominator (Cnd) Error



P4, Confusing the numerator and denominator (Cnd)

R: Yes. Now, 18, what did we say?
"I said the number of bookshelves of, Ali. x is the initial number of books. In x+4, I provided it by saying that he bought four more books afterward. I specified three as the number of books in each library."
R: OK. Well, according to our problem, why is 18 in the numerator, and why is x+4 in the denominator? "For sharing the books in the library…"

P4, Confusing the numerator and denominator (Cnd), Conversation with P4 and researcher

In equations that have algebraic fractional expressions with the numeric numerator and algebraic denominator, 1 confusing the numerator and denominator (Cnd) error has been made regarding only the quotient meaning of the fraction. This error is a type of error that is only seen in this fraction type.

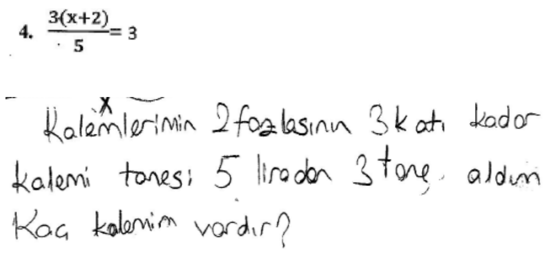
Measurement Meaning

According to the data obtained, one of the other meanings in which the fraction's most mistakes (27,77%) were made was the measurement meaning. There are 3 different types of errors regarding this meaning of fraction. Most mistakes were made in equations with algebraic fractions whose both numerator and denominator are algebraic. The most problems (43,75%) about the meaning of fraction measurement were written in equations with algebraic fractions containing this fraction type.

The most common mistake regarding the meaning of fraction measurement was changing the equation (CE), similar to the fraction meaning of quotient. The meaning of the variable (MV) was another error type that emerged only in terms of fraction measurement meaning. In Figure 2 and Figure 3, there are examples of errors made in this sense of the fraction.

Figure 2.

Examples of Word Problems That are Wrong

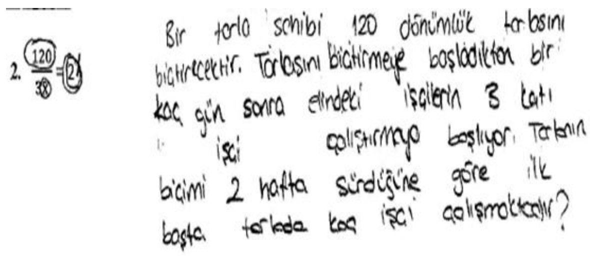


P3, Meaning of the variable (MV)

For instance, P3 was written as the number of items rather than the amount of money in the share component, and we attempted to discover the number of items by dividing by the amount of money. He could not modify the variable's assigned value. However, because he attempted to determine how many pencils he bought by dividing the amount of money by the number of pencils, he employed the meaning of the word "fraction" in his problem.

Figure 3.

Example of wrong word problems



P2, Adding mathematical content (AMC)

Adding mathematical content (AMC) has been determined as an error type that occurs only in the sense of measuring the fraction. P2; In her problem, she divided the field by the amount of work of the workers and tried to express that the field was plowed in 2 weeks and thought to use the meaning of fraction measurement. However, she added a different mathematical content by using the expression for a few days.

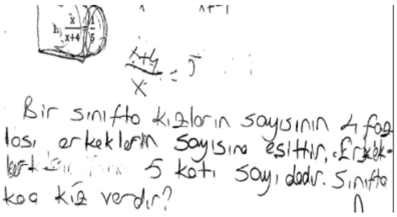
Ratio Meaning

The meaning of ratio has become one of the other meanings of the fraction with an error (16,66%). There are 3 different types of errors regarding this meaning of fraction. The largest number of problems (55,88%) about this meaning of fraction are equations with algebraic fractions whose both numerator and denominator are algebraic. On the other hand, the most errors (83,33%) were seen in problems written to equations with algebraic fractions whose numerator is number and denominator is algebraic.

There are three different types of errors regarding this meaning of fraction. Especially Don't act like a non-fractional equation (Dnfe) has become an error type that occurs in the sense of fraction ratio and division. In Figure 4 and Figure 5, there are examples of errors made in this sense of the fraction.

Figure 4.

Examples of Word Problems That are Wrong

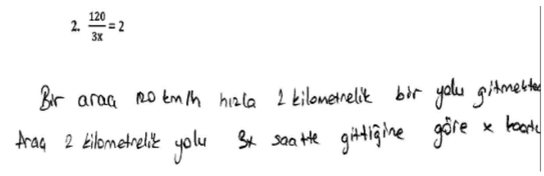


P3, Don't act like a non-fractional equation

P3 used the ratio meaning of the fraction in his problem, but could not emphasize the fraction. The problem he wrote satisfies the algebraic expression "x+4 = 5x".

Figure 5.

Examples of word problems that are wrong



P4, Changing the equation (CE) and Confusing the numerator and denominator (Cnd)

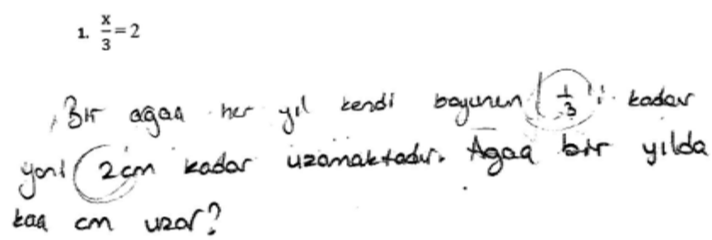
The changing the equation (CE) error has also been one of the errors in the meaning of the ratio. In his problem, P4 expressed the time by proportioning the distance and speed to each other and used the ratio meaning of the fraction. However, in the problem posed by P4, he first changed the equation by expressing two as in the denominator of the fraction and 3x on the opposite side of the equation. On the other hand, he had confusion in the numerator and denominator by mixing the units of speed, distance, and time, and made the mistake of Confusing the numerator and denominator (Cnd).

Operator Meaning

The meaning of operator has become one of the other meanings of the fraction with an error (5,55%). This meaning is only used in equations containing algebraic fractional expressions with algebraic numerators and numeric denominators. There are two different types of errors regarding this meaning of fraction. The Changing the equation (CE) and The no question (NQ) errors occurred in the sense of the operator of the fraction. Figure 6 shows examples of errors made in this sense of the fraction.

Figure 6.

Example of a Word Problem with a No Question (NQ) Error



P4, No question (NQ)

In the problem he wrote, P4 stated that the tree grew by 1/3 of its height and used the operator meaning of fraction. However, he also wrote the answer to the question. Therefore, it is considered that there is no question about the problem.

Part-whole Meaning

No errors were encountered in the verbal problems about the part-whole meaning of fractions.

Conclusion and Discussion

In this study, four main results were reached. The first is that the students make mistakes while posing problems to equations containing algebraic fractional expressions. Oksuz (2004) stated that the transition from fractions to algebraic fractions is a complex process and many misconceptions and misunderstandings make this transition difficult (Oksuz, 2004).

The second result of the research is that the students made the most mistakes in quotient meaning. No study has been found in the literature regarding this result. On the other hand, there are any mistakes in the fraction part-whole. The main reason for this may be related to the fact that students have more knowledge of the part-whole meaning of fractions. Because according to

Perera and Valdemorós (2007) and Dogan-Coskun (2019) students construct the other four meanings of fractions with the help of the part-whole meaning. The part-whole meaning of fraction is mostly emphasized in curricula and textbooks (Eroglu, Camci & Tanisli, 2019). In addition, Kieren (1993) and Lamon (2021) stated that the meaning of fraction, which the teachers in schools most emphasize, is the part-whole meaning.

The third result of the research is that some errors made in problem posing may occur in every sense of the fraction, while some may appear specific to the meaning of the fraction. While it was thought that the errors of changing the notation, not having a question in the word problem, using incomplete data, adding mathematical content, were not related to the meaning of the fraction, mixing the fractional variable with the non-fractional variable, confusion in the meaning of the variable, confusing the numerator and denominator, and unable to associate it with daily life were found to be related to the meaning of the fraction. In addition, the errors made differ according to the meaning of the fraction. For example, while the mixture of numerator and denominator is related to the basic unit of measurement in word problems involving the meaning of fraction measurement, in verbal ratio problems, the ratio of units to each other is in question. For this reason, the errors made in the problems written according to the different meanings of the fraction also differ. No study has been found in the literature regarding this result.

The fourth result is that there are differences in the error cases depending on the type of fraction. For example, Adding mathematical content (AMC) error has only been encountered in equations that contain fractional expressions whose numerator is numeric and whose denominator is algebraic. No study has been found in the literature regarding this result. In future studies, teaching practices can be designed to eliminate these errors and the effectiveness of the applied teaching can be examined.

This study has been restricted to addition and subtraction in fractional algebraic formulas. Different forms of equations or other procedures in algebraic fractional expressions can be researched in future research. The grade level researched may also differ in this situation. The study's use of a holistic case study is another shortcoming. In future study, designs such as experimental design and action research may be utilized.

Etihics Committee Approval: Since this study was produced from the responsible author's doctoral thesis, which was completed in 2018, all permissions have been obtained from the necessary authorities, but there is no ethics committee approval.

Informed Consent: Informed consent was obtained from the participants.

Peer-review: Externally peer-reviewed.

Authors' Contribution: Research study design and implementation, data collection and analysis, drafting the manuscript– M.T.; Critical review for intellectual content and approval of the final version of the manuscript– All authors.

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References

Acar, N. (2010). *Kesir Cubuklarinin Ilkogretim 6. Sinif ogrencilerinin Kesirlerde Toplama ve Cikarma Islemlerindeki Basarilarina Etkisi.* Yayinlanmis Yuksek Lisans Tezi. Selcuk Universitesi, Fen Bilimleri Enstitusu, KONYA.

Akcay, A. O., & Ardic, F. (2020). Sinif ogretmeni adaylarinin kesirlerde problem kurma becerilerinin incelenmesi. *The Journal of International Education Science*, 25(7), 108-119. DOI: http://dx.doi.org/10.29228/INESJOURNAL.47919

Alibali, Martha W., Mitchell J. Nathan, Matthew S. Wolfgram, R. Breckinridge Church, Steven A. Jacobs, Chelsea Johnson Martinez & Eric J. Knuth (2014) How Teachers Link Ideas in Mathematics Instruction Using Speech and Gesture: A Corpus Analysis, *Cognition and Instruction*, 32:1, 65-100, DOI: 10.1080/07370008.2013.858161

Ayllón, M.F. (2005). Invención de Problemas con Números Naturales, Enteros Negativos y Racionales: Tarea para Profesores de Educación Primariaen Formación; Trabajo de investigación tutelada, Universidad de Granada: Granada, Spain

An, S., Kulm, G., & Wu, Z. (2004). The pedagogical content knowledge of middle school mathematics teachers in China and the U.S. *Journal of MathematicsTeacher Education*, 7(2), 145–172.

Aydogdu Iskenderoglu, T. (2018). Fraction multiplication and division word problems posed by different years of pre-service elementary mathematics teachers. *European Journal of Educational Research, 7*(2), 373-385.

Barlow, A.T.; Cates, J.M. (2006). *The impact of problem posing on elementary teachers' beliefs about mathematics and mathematics teaching*. Sch. Sci. Math. 106, 64–73.

Basturk, S. (2016). Primary student teachers' perspectives of the teaching of fractions. *Acta Didactica Napocensia*, 9(1), 35-44.

Birgin, O. ve Gurbuz, R.(2009). Ilkogretim II. Kademe Ogrencilerinin Rasyonel Sayilar konusundaki Islemsel ve Kavramsal Bilgi Duzeylerinin Incelenmesi. *Egitim Fakultesi Dergisi*, XXII (2), 2009, 529-550

Bossé, M. J., Adu-Gyamfi, K. and Cheetham, M. (2011). Translations among mathematical representations: Teacher beliefs and practices. *International Journal of Mathematics Teaching and Learning*, 15(6), 1–23.

Brown, S. I., & Walter, M. I. (2005). *The art of problem posing* (3rd ed.). Lawrence Erlbaum Associates.

Brownell, W. (1935). *Psychological considerations in the learning and teaching of arithmetic*. In The teaching of arithmetic (Tenth yearbook of the National Council of Teachers of Mathematics) (pp. 1–31). New York: Bureau of Publications, Teachers College.

Bunar, N. (2011). *Altinci sinif ogrencilerinin kumeler, kesirler ve dort islem konularinda problem kurma ve cozme becerileri*. Yuksek Lisans Tezi. Afyon Kocatepe Universitesi. AFYON.

Byrnes, J. (1992). The conceptual basis of procedural learning. *Cognitive Development*, 7, 235-257.

Byrnes, J., & Wasik, B. (2009). Factors predictive o f mathematics achievement in kindergarten, first and second grades: An opportunity-propensity analysis. *Contemporary Educational Psychology*, 34, 167-183.

Christou, C.; Mousoulides, N.; Pittalis, M.; Pitta-Pantazi, D.; Sriraman, B. (2005). *An empirical taxonomy of problem posing processes*. ZDM 2005, 37, 149–158.

Cai, J., Hwang, S., Jiang, C., & Silber, S. (2015). Problem-posing research in mathematics education: Some answered and unanswered questions. In F. Singer, N. Ellerton & J. Cai (Eds.), *Mathematical problem posing.* Springer. https://doi.org/10.1007/978-1-4614-6258-3

Chae, J. (2005). *Middle school students' sense-making of algebraic symbols and construction of mathematical concepts using symbols*. PhD Thesis. Indiana University, Graduate Faculty of The University of Georgia in Partial.

Crooks, N. M. & Alibali, M. W. (2014). Defining and measuring conceptual knowledge of mathematics. *Developmental Review*. doi: 10.1016/j.dr.2014.10.001

De Jong, T., & Ferguson-Hessler, M. G. M. (1996). Types and qualities of knowledge. *Educational Psychologist, 31*(2), 105–113.

Dogan-Coskun, S. (2019). The Analysis of the Problems Posed by Pre-service Elementary Teachers for the Addition of Fractions. *International Journal of Instruction*, 12(1), 1517-1532.

Empson, S. B. (1995). Equal sharing and shared meaning: The development of fraction concepts in a first grade classroom. Paper presented at the *American Educational Research Association*, San Francisco, CA.

English, L. D. (1997). The development of fifth-grade children's problem-posing abilities. *Educational Studies in Mathematics, 34*(3), 183-217.

Eroglu, D., Camci, F. ve Tanisli, D. (2019). Altinci sinif ogrencilerinin kesirler ve kesirlerdeki toplamacikarma konusundan bilgilerinin yapilandirilmasina iliskin tahmini ogrenme yol haritasi. *Pamukkale Universitesi Egitim Fakultesi Dergisi*, 45, 116-143.

Garofalo, J. and Lester, F.K., Jr.: (1985). 'Metacognition, cognitive monitoring, and mathematical performance', *Journal for Research in Mathematics Education* 16

Ghasempour, Z., Bakar, N., & Jahanshahloo, G. R. (2013). Innovation in teaching and learning through problem posing tasks and metacognitive strategies. *International Journal of Pedagogical Innovations, 1*(1), 53-62.

Gonzales, N. A. (1994). Problem posing: A neglected component in mathematics courses for prospective elementary and middle school teachers. *School Science and Mathematics*, 94(2), 78–84. https://doi.org/10.1111/j.1949-8594.1994.tb12295.x

Greeno, J. G. (1978). Understanding and procedural knowledge in mathematics instruction. *Journal Educational Psychologist* Volume 12, 1978

Harwood, T. G. and Garry, T., (2003). An overview of content analysis. *The Marketing Review*, Volume 3, Number 4, 1 December 2003, pp. 479-498(20). Westburn Publishers Ltd. DOI: https://doi.org/10.1362/146934703771910080

Hiebert, J., & Carpenter, T. P. (1992). *Learning and teaching with understanding*. In D. A. Grouws (Ed.), Handbook of research on mathematics teaching and learning (pp. 65-97). New York: Mcmillan.

Hiebert, J., & Lefevre, P. (1986). *Conceptual and procedural knowledge in mathematics: An introductory analysis.* In J. Hiebert (Ed.), Conceptual and procedural knowledge: The case of mathematics (pp. 1-27). Hillsdale, NJ: Lawrence Erlbaum Associates.

Isik, C., & Kar, T. (2012). The analysis of the problems posed by the pre- service teachers about equations. *Australian Journal of Teacher Education*, 37(9), 93-113

Izsak, A. (2008). Mathematical knowledge for teaching fraction multiplication. *Cognition and Instruction*, 26(1), 95–143.

Jääskeläinen, R. (2010). Think-aloud protocol. In Y. Gambier & L. van Doorslaer (Eds.), Handbook of translation studies (Volume 1) (pp. 371-373). *John Benjamins Publishing Company*. https://doi.org/10.1075/hts.1

Kavuncu, T. & Yenilmez, K. (2021). Besinci sinif ogrencilerinin kesir modellerine uygun problem kurma ve cozme becerilerinin incelenmesi. *Eskisehir Osmangazi Universitesi Turk Dunyasi Uygulama ve Arastirma Merkezi (ESTUDAM) Egitim Dergisi*, 6 (2), 201-218.

Kieren, T.E. (1993). Rational and fractional numbers: From quotient fields to recursive understanding. *In Rational Numbers: An Integration of Research*; Carpenter, T.P., Fennema, E., Romberg, T.A., Eds.; Erlbaum: Hillsdale, NJ, USA, pp. 49–84.

Koichu, B., Harel, G., & Manaster, A. (2013). Ways of thinking associated with mathematics teachers' problem posing in the context of division of fractions. *Instructional Science, 41*(4), 681-698.

Kojima, K., Miwa, K., & Matsui, T. (2009). Study on support of learning from examples in problem posing as a production task. In S.C. Kong et all. (Eds.). *Proceedings of the*

17th International Conference on Computers in Education [CDROM]. Asia-Pacific Society for Computers in Education.

Kopparla, M., Bicer, A., Vela, K., Lee, Y., Bevan, D., Kwon, H. & Capraro, R. M. (2019). The effects of problem-posing intervention types on elementary students' problem-solving. *Educational Studies*, 45(6), 708-725. https://doi.org/10.1080/03055698.2018.1509785

Lamon, S.J. (2001). Presenting and representing: From fractions to rational numbers. In The Roles of Representation in School *Mathematics*; National Council of Teachers of Mathematics: Reston, VA, USA; pp. 146–165

Lesh, R. and Doerr, H. M. (2003). *Foundations of models and modelling perspective on mathematics teaching, learning, and problem solving.* R. Lesh and H. M. Doerr (Eds.), Beyond constructivism: Models and modelling perspectives on mathematics problem solving, learning, and teaching in (pp. 3–33). Mahwah: Laurence Erlbaum.

Martinez, S.; Blanco, V. (2021). Analysis of Problem Posing Using Different Fractions Meanings. *Educ. Sci.* 2021, 11, 65. https://doi.org/ 10.3390/educsci11020065

Merriam, S. B., & Tisdell, E. J. (2016). *Qualitative research: A guide to design and implementation* (4th ed.). John Wiley & Sons.

Oksuz, C. (2004). *Children understanding of algebric fraction as quotients*. (Unpublished doctoral dissertation). University of Arizona, Arizona

Panasuk, R. M., & Beyranevand, M. L. (2010). Algebra students' ability to recognize multiple representations and achievement. *International Journal for Mathematics Teaching and Learning*, 1–21.

Pardhan, H. & Mohammad, R.F., (2005). Teaching Science and Mathematics For Conceptual Understanding? *A Rising Issue Eurasia J. Math. Sci. & Tech*. Ed., 1(1), 1-20.

Perera, P.B.; Valdemoros, M.E. (2007). Propuesta didáctica para la enseñanza de las fracciones en cuarto grado de educación primaria. *In Investigación en Educación Matemática XI*; SEIEM: San Cristóbal de la Laguna, Tenerife, pp. 209–218.

Pirie, S. E. B. (2002). *Problem posing: What can it tell us aboutstudents' mathmatical understanding*. In Proceedings of the *24th Annual Meeting North American Chapter of the International group for the Psychology of Mathematics Educatio* (pp. 925-958). GA, Athens.

Ploger, D., & Hecht, S. (2009). Enhancing children's conceptual understanding of mathematics through Chartworld software. *Journal of Research in Childhood Education*, 23(3), 267-277.

Robinson, K. M., & Dubé, A. K. (2009a). Children's understanding of addition and subtraction concepts. *Journal of Experimental Child Psychology, 103*, 532–545.

Roth,K, Jones,B, Idol,L, (1990). *Developing meaningful conceptual understanding in science*. Dimensions of thinking and cognitive instruction1990Hilldale, NJErlbaum139175

Silver, E. A. (1994). On mathematical problem posing. *For the Learning of Mathematics*, 14(1), 19–28.

Stephens, M. (2003). *Regulating the entry of teachers of mathematics into the profession: challenges, new models, and glimpses into the future*. Second International Handbook of Mathematics Education pp. 767-793

Star, J. R. (2001). *Re-conceptualizing procedural knowledge: Innovation and flexibility in equation solving*. Unpublished doctoral dissertation, University of Michigan, Ann Arbor.

Stoyanova, E. (1998). *Problem posing in Mathematics Classrooms*. In Research in Mathematics Education: A Contemporary Perspective; McIntosh, A., Ellerton, N., Eds.; Edith Cowan University, MASTEC: Perth, WA, USA; pp. 164–185.

Tastepe, M. & Yanik, H. B. (2021). Kavramsal bilginin gelisiminin incelenmesi: Cebirsel kesirli ifadeleri iceren denklemler baglaminda. *Uluslararasi Sosyal ve Egitim Bilimleri Dergisi*, 16: 83-103.

Toluk, Z. (2001). Esit paylasim ortamlarinin kesir ogretiminde kullanimi. *Kuram ve Uygulamada Egitim Bilimleri*. 1 (1)

Van de Walle, J.A. Karp, K.S. ve Bay-Williams, J.M. (2012). *Ilkokul ve ortaokul matematigi: Gelisimsel yaklasimla ogretim*. (Cev. Editoru: Soner Durmus). Ankara: Nobel Yayin Dagitim. 7. Basimdan Ceviri.

Yildirim, A. & Simsek, H. (2013). *Sosyal bilimlerde nitel arastirma yontemleri*. (9. Baski). Ankara: SeckinYayincilik.

Yin, R. K. (2018). *Case study research and applications: Design and methods* (6th ed.). Sage Publications.

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