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Sixth Grade Students' Some Difficulties and Misconceptions on Angle Concept

Deniz OZEN UNAL*

Omer URUN**

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Abstract: This study aims to determine the misconceptions and difficulties of sixth-grade students on the subject of angles. The study participants are 25 sixth grade students from a public school in a city in western Turkey during the 2017-2018 academic year. This qualitative study used 17 open-ended questions designed by the researchers for data collection to examine the students' misconceptions and difficulties. Data were examined by implementing content analysis. It has been analysed that students cannot define the angle due to difficulties and misconceptions in determining the corners and edges of a symbol. Besides, they also find it difficult to compare the measures of the angles, adjacent angles, complementary and supplementary angles.

Keywords: Angle concept, misconception, student difficulties

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* Correspondence: Aydın Adnan Menderes University, Turkey. deniz.ozen@adu.edu.tr

** Nazilli Science and Art Center (BILSEM), Turkey, omerurun@gmail.com

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Introduction

The concept is defined as "the general design encompassing the common features of objects or events and consolidate them under a common name" (Turkish Language Association, 2018) in mathematics. The concept can become more meaningful by seeing the relationships and transitions among other concepts in addition to merely recognizing and knowing them (Baki, 2015). Besides, the concepts are interrelated in mathematics, and the students act on the inferences by attaching different meanings to a concept. Such unconscious mistakes in the learning process can create misconceptions (Turkdoğan et al., 2015).

A misconception is expressed as a person's interpretation of a subject that makes sense to self yet in a way that contradicts the conceptual understanding of experts in that field (Baki, 2015). Misconception should be considered as producing an incorrect answer due to an error or lack of knowledge and a developed perception or conception that is diverged from expert opinion (Hammer, 1996) that causes systematic errors (Smith III et al., 1993). Misconceptions are accepted as unconscious behaviours of students with no mathematical validity, resulting from false beliefs and experiences (Baki, 2015). If students confidently explain why their mistakes are correct, it can be said that there is a misconception and it is challenging to address these misconceptions (Yenilmez & Yasa, 2008). This difficulty stems from the fact that deeply rooted misconceptions and the obstacle to conceptual understanding (Minstrell, 1982) are supported by the experiences of individuals; thus, they constantly resist change (Cox & Mouw, 1992). Considering the interconnected nature of mathematics, having misconceptions about prior knowledge could lead to inaccurate learning for students and more deeply rooted misconceptions (Baki, 2015; Driver & Easley, 1978; Sandir et al., 2007; Woodward et al., 1994). Misconceptions are expressed as incorrect meanings based on misunderstandings and misinterpretations (Ojose, 2015) and should not be addressed only within student failure and errors (Graeber & Johnson, 1991). Also, the teaching method used could be influential in forming misconceptions (Zembat, 2010). It is natural to experience misunderstanding in mathematics; therefore, possible misconceptions about concepts must be noted so that methods to remediate such misconceptions can be developed to avoid persistence (Ojose, 2015). The necessity of providing comprehensive mathematical concept knowledge for conceptual learning has made it valuable to seek solutions to identify and remediate misconceptions and address the lack of knowledge (Kucuk & Demir, 2009). Majority of educational studies are focusing on the identifying students' lack of knowledge and efforts to address them (Akbulut & İsik, 2005; Akkaya & Durmus, 2006; Ay, 2017; Ayyildiz & Altun, 2013; Basturk & Donmez, 2011; Keceli & Turanli, 2013; Simon, 2006; Ojose, 2015; Temel & Eroglu, 2014; Turkdogan, Guler et al., 2015; Turnuklu et al., 2013; Yenilmez & Uysal, 2007). This study also aimed to identify sixth-grade students’ difficulties and misconceptions about angles.

Conceptual Analysis of Studies on Student Difficulties and Misconceptions About Angles

Geometry is an essential component of mathematics education students find difficult to understand (Van Hiele, 1986). Understanding the fundamental geometric concept of angle is essential for learning about this component from an early stage (French, 2004). However, the angle is one of the oldest and fundamental concepts in which students lack knowledge and misconceptions. Of basic geometry concepts, it is identified as one of the most abstract and the most challenging concepts for students (Tanguay & Venant, 2016). This undesirable situation makes it difficult to advance in other subjects of geometry (Moore, 2013). Studies on the difficulties and misconceptions related to angles were reviewed to identify their common aspects and document reported misconceptions (see Table 1). "Angle definition and types", "angle construction", "notation of an angle with the symbol", "angle measure", "adjacent, complementary, supplementary and opposite angles" and "constructing a perpendicular line from a point on the line or not on the line" categories were used to classify the errors and misconceptions based on this review.

Table 1.

Misconceptions about Angles in the Literature

Errors and Misconceptions	Researchers
Related to Angle Definition and Types:	
• Not being able to define, not having a sufficient language to define, difficulty in expressing it verbally, • Reference to the "corner" of a physical object or geometric shape, • Emphasis on "measure" in angle definition, • Expressions including types of angles • Not recognizing the right angle when its edges are not parallel to the edges of the paper, • Obtuse angles are not seen as an angle, • Straight angles are not noticed, • Definitions were usually incorporating expressions that are visually formed in the minds and include other geometric concepts such as "The distance between the sides of a polygon; The distance between two intersecting lines; The measure between two points starting from common endpoint; The degree between the rays of two intersecting line segments; A ray extending along a straight line; It is a unit of measure."	*Baldy et al. (2005), Butuner & Filiz (2017), Cetin & Dane (2004), Dane & Baskurt (2011), Doyuran (2014), Erbay (2016), Keiser (2004), Kilic, Temel & Senol (2015), Taylan & Aydin (2017), Yesildere (2007).*
Related to Angle construction:	
• The obvious superiority of the right angle as a prototype and the learning obstacle, • Representing the vertex of a physical object or geometric figure and its effect in drawings • One edge of the angle must be horizontal.	*Baldy et al (2005), Devichi & Munier (2013), Doyuran (2014), Mitchelmore (1998).*

Related to notation of an angle with symbol: • $Notations\ \ such\ as\ \widehat{AB}, AB, ABC, (\widehat{ABC})$	Doyuran (2014)
Related to Angle Measure: • It depends on the size of the shaded interior region, • Depends on the length of the arc (or radius of the arc) pointing to the angle, • It depends on the length of the edges and there may be a right-angle effect in this misconception, • Congruent angles with different orientation or edge lengths are different angles.	Butuner & Filiz (2017), Devichi & Munier (2013) Keiser (2004), Mitchelmore (1998).
Adjacent, Complementary, Supplementary and Opposite Angles: Insufficiency in defining these angles • Inability to identify these angles in a shape, • Expressing complementary angles as "90°", supplementary angles as "180°", • Only angles with one opposite edge are considered opposite angles.	Erbay (2016), Taylan & Aydin (2017).
Related to drawing a perpendicular from a point on the line or not on the line: • The tendency of positioning the perpendicular to be drawn from a point not on a line parallel to the long side of the paper, • A perpendicular cannot be drawn from a point on a line.	Butuner & Filiz (2017).

The existence of various definitions of the concepts in the literature is shown as one of the most important reasons for students' difficulties (Butuner & Filiz, 2017; Henderson & Taimina, 2005; Keiser, 2004). Definitions as the building blocks of mathematical thoughts undertake a fundamental task in forming a concept and distinguishing it from other concepts (Cakiroglu, 2015). According to Cakiroglu (2015), although there are many ways to define concepts clearly, expressions that do not meet the definition criteria will confuse, and mathematical communication could not be ensured. According to Keiser (2004), depending on students' concept use in mathematics or other sciences, what makes students' learning of particular concepts even more complicated is that some concepts continue to change in their emphasis; therefore, the definitions change over time.

Angle is such a concept that has been defined differently over the centuries. Depending on the mathematical context, the concept of angle can even carry different meanings today. Although the angle concept is perceived as simple, it is a multi-faceted concept, making it possible to have various definitions throughout history. However, definitions differ significantly in their emphasis.

Keiser (2004) classified the definitions of the concept of angle as

- A measure of the turning of a ray about a point from one position to another (dynamic),
- the union of two rays with a common endpoint (static)
- the region contained between the two rays (static) (p.288).

She stated that the definitions could generally include one of the three. Another classification is defined angle with the different perspective as “a geometric shape”, “a changing and dynamic structure”, and “a measurable attribute” (Henderson & Taimina, 2005). In these perspectives, the dynamic concept of angle includes action in the form of a rotation, rotation point, or direction between two lines. The angle as a measure is explained as the arc length, or the ratio between the areas of the circle segments and angle as a geometric shape is explained by two lines intersecting in space (Henderson & Taimina, 2005).

These definitions limit the concept by focusing more on one aspect of the angle regarding a relation, a quality, or a quantity than the other (Keiser, 2004). In the definitions, angle and angle measure concepts were used in the same meaning (Ertekin, 2015), this made it unclear what precisely the angle is, which situation indicates an angle, and what exactly was measured while measuring an angle (Henderson & Taimina, 2005; Keiser, 2004) and the necessary transitions between definitions (Cakiroglu, 2015) almost impossible. The first classification as "a measure of the turning of a ray about a point from one position to another" may correspond to the direction angle concept in trigonometry. Depending on the direction, it is possible to shift between definitions of the angle. It is formed with the ray in its pre-rotation initial position and the ray in its post-rotation ending position. Moreover, it is formed by the union of two rays with a common endpoint (Ertekin, 2015). In this case, the static angle structure will result from the situation representing the dynamic angle (Mitchelmore, 1998). If the angle is defined as the region contained between two rays, the region's size can be considered the measure of the angle, yet considering the infinity of the rays, the measurement will not be possible (Ertekin, 2015). Similarly, one could see that if the distance between the rays forming the angle is considered as the measure of the angle, it is not meaningful to measure it linearly. The length between the selected points on the lines would differ based on selection, although the rays' width remains the same (Kabaca, 2015).

Keiser (2004) observed similarities between the student definitions of angle concept and definitions or explanations recorded since Euclid's time when she examined the data obtained from student comments only with a historical perspective. This multifaceted structure, which manifests itself in defining the concept of angle, affects students' perceptions of the concept and results in misconceptions. There are several pieces of evidence to demonstrate this situation for students. The studies showed that the students had difficulties focusing on an angle in a triangle or square (Mason, 1989) by not defining the angle and the triangle adequately (Mason, 1989). Many students thought that an angle measure depends on the radius of the arc that points the angle and that the measure of the angle is related to the edge lengths (Devichi & Munier, 2013) or had the belief that one edge of the angle should be positioned horizontally. The direction should always be counterclockwise (Mitchelmore, 1998). Ubuz (1999) specifically emphasized that the reasons for 10th and 11th-grade students’ errors and the misconceptions that cause these errors were the same in almost every question, and stated that this situation is a result of students are at the first level- visual- of the van Hiele geometry thinking levels. The studies on the subject conducted with the sixth and seventh-grade students showed that,

- students could not interpret the verbally written angle definition mathematically,
- although it is not known how many regions the angle separates the plane into, this topic is not cover with a complete understanding of this concept since students being able to find the points in the inner and outer regions,
- students do not know the concepts of angle and angular region,
- students' inability to draw the right angle and the reflex angle,
- students could not construct the meaning for angle edge concept,
- students could not write the common and non-common sides of adjacent angles,
- the idea that supplementary angles should always be adjacent reveals misconceptions that the concepts of the opposite, alternate interior, alternate exterior and congruent angles cannot be represented in figures (Doyuran, 2014; Erbay, 2016 as cited in Ozbellek, 2003).

In addition, studies have been conducted on the process of internalizing the concept of angle by primary school students (Keiser, 2004), examining the relationship between the edges length of an angle and the measure of the angle in this process (Devichi & Munier, 2013), the difficulties experienced by middle school students in understanding the line segment, linearity, ray and angle (Dane & Baskurt, 2011). It has been stated that a right angle can be a significant learning barrier, in particular for the tasks asked to focus on the edge length and angle (Devichi & Munier, 2013). On the other hand, Butuner and Filiz (2017) showed that most high-achieving sixth-grade students have misconceptions and have difficulties understanding the conceptual meaning of angle concept.

The reasons for the difficulties experienced by students in the learning process are stated in the sources, including the natural structure of the concept (epistemological obstacle) in its historical development, students' readiness level, comprehension ability (genetic and psychological obstacles) for personal development, and the obstacles deriving from the nature of learning, teaching model or the way the subjects are introduced in the sources (pedagogical (didactic) obstacle) (Cornu, 1991). Like the discussions in the historical process about the development of angle concept, the situations such as the difficulties arising from the nature of the concept (Keiser, 2004), the inadequacy of students' prior learning (Dane & Baskurt, 2011; Mitchelmore, 1998; Ozbellek, 2003; Van Hiele, 1986), the content and teaching method (Devichi & Munier, 2013; van Hiele, 1986; Zembat, 2010) that may result in misconceptions in students.

Fischbein (1993) examined the geometric reasoning process with a cognitive approach different from a developmental approach (van Hiele, 1986), in his work called The Theory of Figural Concepts, build this process on the combination of the concept and the figure (image) formed with concept's spatial features in mind. According to the theory, the interaction of concept and figure is essential. While the figure helps to make predictions for the solution with the help of intuitions, the concept creates the mathematical foundations of the ideas revealed by the intuitions and ensures that they are consistent. According to Fischbein (1993), the high-level reasoning process is the situations in which the concept manages the figure and then these two components transform into formal concept knowledge. Situations where the reasoning process is under the control of the figure will be an essential source of student errors and difficulties that will cause students to make incorrect inferences. The development of this process can be achieved by designing learning environments that strengthen students' conceptual knowledge and allow effective interaction between figure and concept. In this respect, several researchers have stated the necessity and importance for teachers to be aware of students' difficulties and misconceptions during instruction planning (Cornu, 1991; Graeber, 1999; Stump, 2001). In addition, this necessity and importance were discussed within the scope of pedagogical content knowledge (Shulman, 1986). It was deemed necessary for teachers to know and implement approaches to help students overcome these difficulties during the teaching process (Graeber, 1999; Ojose, 2015). However, studies show that teachers cannot anticipate the misconception and its root causes (Gokkurt Ozdemir et al., 2017). They cannot identify possible misconceptions, and they do not see that they may prevent students' learning in further practice (Asquith et al., 2007). The examination of the studies showed that misconceptions can take place at various grade levels. The fact that mathematics teaching in our country is mainly based on procedural knowledge makes it difficult to change this habit in the future, to adopt the conceptual view of mathematics for pre-service teachers, and to balance conceptual and procedural learning in their professional lives (Baki, 2015).

Angle Concept in Mathematics Curriculum

In the Mathematics Curriculum (Ministry of National Education [MoNE], 2013, 2018), the angle concept is firstly introduced in the sub-learning area of basic concepts in geometry in the third grade. Although the primary objectives are the same, some of the objectives have changed across grade levels in the 2018 mathematics program (MoNE, 2018). For example, the objective of "draws a perpendicular from a point on or outside a line" was in the 6th grade in 2013 program and moved to the 5th grade in the 2018 program (MoNE, 2013, 2018). Students' expressing the concepts of point, line, line segment and ray and giving examples of angles from their surroundings are firstly introduced in the 2018 Mathematics Curriculum (MoNE, 2018). In the fourth grade, the objective of 'determining of the rays forming the angle and the corner, naming the angle and showing it with a symbol' was included. This grade level includes the measure of the angle and the classification of the angles, the awareness that the angle is formed by the rotation of a ray around the starting point, and the explanations that the difference in the positions of the angles with the same measure does not influence the measure of the angle.In the fifth grade, the concept was handled in the geometry and measurement learning domain, and the students were asked to form acute, right and obtuse angles on grid paper and determine them. The instructions about drawing perpendiculars to a line from a point or not on a line are also included. In the sixth grade, in addition to the objective of "knows that the angle is formed by two rays with the common end point and shows it with a symbol" ; “draws a congruent angle of an angle” and “explores the properties of complementary, supplementary and opposite angles; and solves related problems" objectives are included.Thus, the curriculum includes two definitions that ease the transition between angle as "a measure of the turning of a ray about a point from one position to another (dynamic angle)" and "the union of two rays with a common endpoint (static)" (Ertekin, 2015). The width between the edges of the angle is called the rotational movement by one ray until it overlaps with the other. This width is considered as the measure of the angle, and the mitre and more sensitive protractor tools were determined as standard measurement tools to measure this magnitude by using the basic angle measurement approach (Kabaca, 2015), which determines the number of parts remaining in the angle by splitting the full circle a certain number of times.

This study analyzes the subject's place in the Mathematics Curriculum with a study on literature to identify the challenges and misconceptions about angles. While some of the studies focusing on the scope of student difficulties or misconceptions devoted a section to angle concept and basic geometric concepts, the other part focused on some components. In this study, the concept was examined by considering the Mathematics Curriculum (MoNE, 2013, 2018). In addition, identifying the difficulties faced by students and possible misconceptions is also essential in planning the teaching that will enable them to overcome these difficulties. Therefore, this study aimed to identify the difficulties and misconceptions of sixth grade students regarding the concept of angle. Thus, this study seeks to answer the research question of "What are the difficulties and misconceptions that sixth-grade students have about angle subject?".

The sub-problems of the research are considered with following dimensions.

- What are the difficulties and misconceptions that sixth-grade students have about the definition, drawing and symbolic representation of an angle?
- What are the difficulties and misconceptions that sixth-grade students have about specifying the vertex and edges of an angle?
- What are the difficulties and misconceptions that sixth-grade students have about the measure of the angle?
- What are the difficulties and misconceptions that sixth-grade students have about adjacent, complementary, supplementary, and opposite angles?
- What are the difficulties and misconceptions that sixth grade students have about drawing a perpendicular line from a point on or not on a line?

Method

The study design was a qualitative case study. The case study is a research method used to answer how or why questions about cases where research focuses on a current phenomenon and where the researcher has almost no control over the events (Yin, 2014). In addition, "what" question is also necessary for the case study and is addressed within this scope (Yildirim & Simsek, 2016). This study aimed to identify the misconceptions and lack of knowledge that sixth-grade students have regarding the angle concept, one of the foundational concepts of geometry.

Participants

The participants of this research are 25 sixth-grade students studying at a public school in a western province of Turkey. A purposeful sampling method of criterion sampling was used in the participant selection process. The inclusion criteria for study participants were (1) being a sixth-grade student, (2) knowing the angle subject, and (3) volunteering for participation. The sixth-grade students were chosen in the study because the concept of angle, which is one of the essential concepts of geometry and handled at the visual level in the curriculum at the beginning, was included in the sixth-grade curriculum together with its formal definition. It is necessary to identify students' misconceptions and difficulties regarding angle concept so that these difficulties and misconceptions can be eliminated before they become more profound and cause difficulties in the further subjects of geometry. All of the participating students were selected from the same school. Seven of them are girls, and 18 are boys, with an average age of 11.

Data Collection

To determine the misconceptions and knowledge gaps of the students about angles, firstly, the studies on this subject in the literature were examined, and knowledge gaps, errors and misconceptions mentioned in the studies related to angle concept and angles were determined. The misconceptions identified are shown in Table 1 of the previous section and were used to develop the open-ended measurement tool.

After examining the studies, researchers developed a measurement tool including 17 open-ended geometry questions that do not require any calculations and suitable for the objectives in the Middle School Mathematics Lesson (5, 6, 7, 8th Grades) Curriculum (MoNE, 2013). This tool included open-ended items to reveal errors or misconceptions on the definition of an angle, such as its drawing, its notation with symbols, the measure of an angle, adjacent, complementary, supplementary and opposite angles, drawing a perpendicular from a point on or not on a line to a line (see Table 2).

Table 2.

The Distribution of the İtems in the Open-Ended Measurement Tool

Content	Item numbers
Angle definition	1, 5
Angle construction (drawing)	1, 3
Angle's vertex, edges and representation by symbol	1, 2, 3, 4
Angle measure	6, 7, 8, 17
Adjacent, complementary, supplementary and opposite angles	9, 10, 11, 12, 13, 14
Drawing a perpendicular from a point on or not on a line	15, 16

The students were given a grid paper and were asked to compare the angles' measurements and respond with drawings. Thus, it was ensured that the students were able to answer without needing any measurement tool (protractor, etc.), and the students were asked to explain their answers and they were provided opportunities to reflect their ideas. The students were asked to draw to compare the measurements of the angles to find out what they focus on. Students were given rulers to use for drawing. The measurement tool included more than one item to find out the misconceptions. Situations that might lead to a misconception were also addressed in the replies. Thus, it has been tried to minimize the influence of situations such as student mistakes caused by carelessness, lack of knowledge or random answers by looking for evidence in such situations. Some students were asked to answer the same questions again to verify the accuracy of the data. In so doing, misconceptions were identified.

The designed items were examined by three experts, two of whom are mathematics educators, and expert opinion was obtained. A sixth-grade student conducted a pilot study, and feedback was received to test whether the items were communicating. Revisions were made in line with the feedback received after the examination, and then the test was finalized by obtaining a second round of expert opinion. Example open-ended items used in the research are given in Appendix 1.

The data were collected after students learned the subject of angles in the 2017-2018 academic year. After administering the open-ended measurement tool, unstructured interviews were conducted with some students to clarify the situations caused by misunderstandings or lack of expression in the items and verify the data. In these interviews with the students, we focused on the incorrect and inconsistent answers and the situations that could be misconceptions were tried to be identified/verified.

Data Analysis

In this study, data obtained from 25 students were analyzed to identify common student difficulties and misconceptions. Content analysis was used with the measurement tool consisting of open-ended problems. With content analysis, it was aimed to describe the data from the students' written responses and reveal the meaningful patterns that may be hidden in the data (Yildirim & Simsek 2016). A literature study on the themes for the difficulties and misconceptions about the angles were used to analyze data. The findings were reported accordingly. These themes are related to students' misconceptions and difficulties regarding the definition and types of an angle, drawing an angle, symbolizing an angle, measuring an angle, adjacent, complementary, supplementary and opposite angles, and drawing a perpendicular from a point on or outside of a line.

Two independent coders coded the data to meet the reliability criteria. Coders added additional codes by considering misconceptions and knowledge gaps in the literature. The disagreements between the two coders were discussed, and the opinion with the highest percentage was considered the common opinion (Lincoln & Guba, 1985). The situations in which both coders made a joint decision were identified as misconception and student difficulties. The codes obtained were presented by being themed based on the subject content. A field study was carried out with the participant students for a sufficient time to ensure the research validity. When the responses were not understood clearly during the interviews, member checking and in-depth descriptions were used in the analysis.

Findings

Findings Regarding Definition, Drawing and Symbolic Representation of Angle

Within the scope of this theme, students were asked to define the angle, draw an angle and represent it with a symbol. However, the findings related to students' difficulties encountered in the literature as they did not have sufficient language to express the concepts and thus, they have a hard time expressing them verbally (see Table 1). Another question discussed the literature expressions on the definition of the angle with visual support.

When the responses given by the students regarding the definition of the angle are examined, only six students (S9, S14, S16, S19, S21, S22) could define the angle as "An angle is a shape formed by two rays with a common endpoint (starting point)". However, it is noteworthy that some of the students who defined the angle in this way responded to yes when they were asked, "Is the angle the region between two rays with the common endpoint?" (S19, S22) and "Is the angle wideness between two rays with the common endpoint?" (S9, S16, S19, S22). Various responses given by 19 students other than these students to the definition of angle are summarized in Table 3.

Table 3.

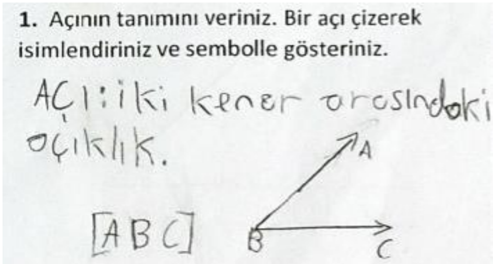
Student Responses Regarding the Definition of Angle

Codes	Example Expression	Students
Common Definition	*"An angle is a shape formed by two rays with a common endpoint"*	S9, S14, S16, S19, S21, S22
Wideness between edges	*"Angle is the wideness between two rays with the common endpoint."*	S1, S5, S6, S11, S12, S15, S17, S24
Other		
Explanations on the measure of the angle or the types of angles	*"Angles are right angles, acute angles, obtuse angles, reflex angles, and straight angles."*	S7, S13, Ö20
Expressions to describe the angle	*"Angle is a varied symbol with edges."*	S3, S18, S25
Expressions made using example	*"We can see an angle in the minute hands of a watch"*	S4
No explanation	---	S2, S8, S10, S23

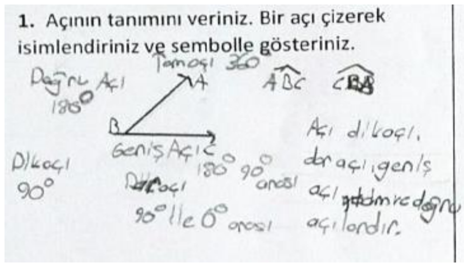
As seen in Table 3, most (f=8) responses to the question (define an angle, draw, name an angle and represent it with symbols) were incorrect. When the definition of the angle was asked, they directly emphasized the width between the edges of the angle (S1, S5, S6, S11, S12, S15, S17, S24) (Figure 1.a) and said: "An angle is a width between two rays with the common endpoint." When asked about the definition of an angle, some students replied with angle measures or types of angles instead (S7, S13, S20). For this, S20's explanation of "*Angles are right angles, acute angles, obtuse angles, reflex angles, and straight angles* "were coded as explanations on the types of angles Figure 1.b). Some of the students used various expressions to describe the angle (S3, S18, S25). One of the remarkable ones was S3's expression, "An angle is a varied symbol with edges." (Figure 1.c). One of the students (S4) explained the angle concept with an example:" *"We can see an angle in the minute hands of a watch."* Finally, four students (S2, S8, S10, S23) did not respond to the open-ended question about the definition of the angle.

Figure 1.

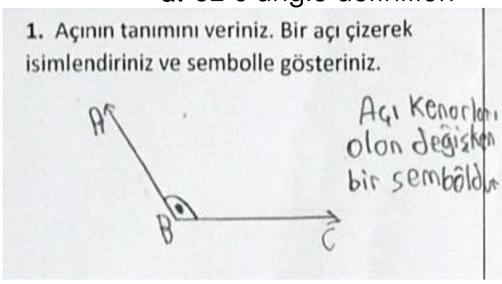
Examples of Students' Responses to the Definition of Angle



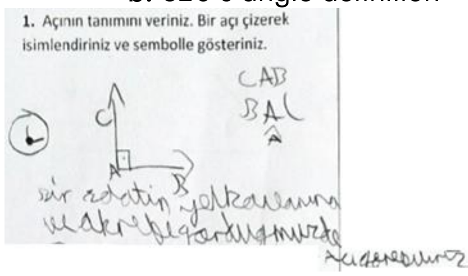
a. S2's angle definition



b. S20's angle definition



c. S3's angle definition



d. S4's angle definition

As seen in Figure 1, when the students were asked to draw angles, it was seen that they could draw various angles. Among these choices, students mostly prefer to draw acute angles and right angles (f=9). Some (f=5) prefer to draw obtuse angles, and a few prefer to draw all types of angles, including acute angle, right angle and obtuse angle (f=1). In addition, it was observed that one student did not draw at all in this question.

When the student responses (Questions 1 and 4) regarding the representation of the angle with a symbol were examined, it was seen that seven students (S6, S11, S12, S17, S19, S20, S24) correctly used symbols either for the angle they drew or for the angle with the given edges. Based on the responses examined under this theme, it was determined that most of the students (S1, S2, S3, S4, S5, S7, S8, S10, S13, S22, S23) had errors regarding the representation of the angle with symbols. It is noteworthy that even if students can correctly identify the vertex of the angle, they have errors in their symbolic representations. In particular, example symbolic representations of S1 and S2 are given in Figure 2.a and Figure 2.b, respectively.

Figure 2.

Examples of Students' Misconceptions About the Angle Symbol

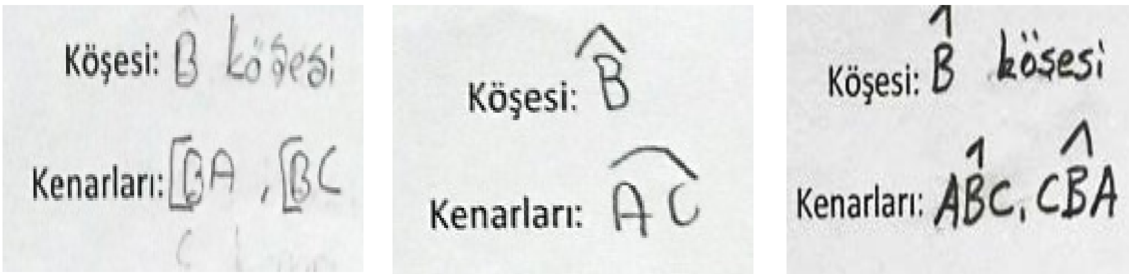
a. S1's symbolic representation **b.** S2's symbolic representation

Findings Regarding Specifying the Corner and Edges of An Angle

Regarding this theme, students were asked to "specify the vertex and edges of $\widehat{ABC}$" (without a given figure). Students' encountered difficulties and misconceptions regarding specifying the vertex and edge of an angle is shown in Table 4. In this question, only three students (S16, S20, S24) stated that the edges of the angle are rays and represented them with symbols (Figure 3.a). Apart from these students, nine students (S3, S6, S8, S10, S12, S15, S20, S21, S24) tended to express vertex or edges as angles (Figure 3.b and Figure 3.c). In addition, 16 students expressed the vertex of the angle as "B" and did not state that it was a point. In the interviews conducted to verify this situation, it was seen that the students could not express it as point B.

Figure 3.

Students' Responses to the Vertex and Edges of an Angle



a. S16's response **b.** S3's response **c.** S10's response

Figure 4.

Examples of Students' Incorrect Drawing of the Angle

a. S13's response **b.** S17's response

In another question, students were asked to draw (KLM)^ using the points K, L and M on grid paper. In this question, two of the students (S3, S13) drew angles incorrectly (Figure 4.a), three students (S11, S17, S25) drew the edges as straight segments instead of rays despite positioning the angle correctly (Figure 4.b). The rest positioned the angles correctly. Here, it was seen that the majority did not have any difficulties in drawing the angle given with its symbol.

Table 4.

Difficulties and Misconceptions in Specifying the Vertex and Edges of an Angle

Codes	Students	f
Ability to specify the vertex and edges of an angle	S16, Ö20, S24	3
The tendency to express vertex or edges as an angle	S3, S6, S8, S10, S12, S15, S20, S21, S24	9
Incorrect drawing of an angle	S3, S13	2
Drawing the edges of an angle as line segments	S11, S17, S25	3

Findings Regarding the Angle Measure

Regarding this theme, four questions asked to the students to elicit the existing students' errors and misconceptions of whether the measure of the angle changes depending on the length of its arms (Figure 5, Figure 6, Figure 7, Figure 11), whether it changes according to the size of the interior region (Figure 10), whether it changes according to the length of the arc that indicates the measure (Figure 8, Figure 10). Students' difficulties and misconceptions regarding angle measurements are shown in Table 5.

Table 5.

Difficulties and Misconceptions about ahe Measure of an Angle

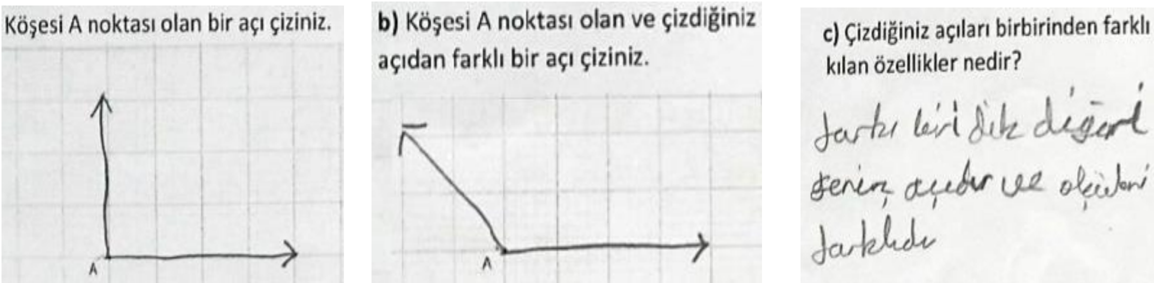
Codes	Students	f
Depends on the length of the arms	S5, S6, S16, S18, S21, S22, S25	7
Depends on the size of the interior region,	S5, S10, S22	3
Depends on the length of the arc pointing to measure	S1, S4, S5, S8, S10, S13, S15, S18, S20, S22	10

Students were first asked to draw an angle and then a different angle with the same vertex, and then they were asked to indicate the features that made these angles different from each other (Figure 5).

In literature, there are similar approaches with the findings (see Table 1) that the angles with the same measure, but different position are not congruent. The angle measurements depend on the edge length, such as the length of the arc pointing to the measure. In so doing, the angle measure can be determined despite the positional change. When the findings related to this question were examined, it was seen that all students except two students (S5, S23) were able to draw the two desired angles in the question. These students gave responses such as "wideness between the edges" (S1, S6, S9, S11, S13, S14), "measures" (S2, S7, S10, S12, S18, S19, S22) or "types" as the features that make the two angles different from each other. (S2, S3, S4, S7, S8, S13, S15, S17, S18, S20, S21, S24, S25). For example, S18 emphasizing the angle measure and types replied as "the difference is one is a right angle, and the other is an obtuse angle, and its measures are different" (Figure 5.c).

Figure 5.

S18's Response



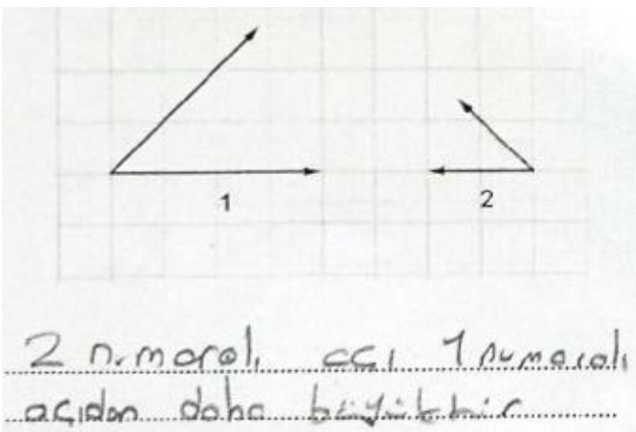
a. b. c.

In another question related to this theme, two congruent angles with different edge lengths, three angles with different measures and the smaller one with the edges drawn long, two angles with the longer length of the arc pointing to the measure and the smaller ones with wider shaded interior region compared, the larger one was given. The students were asked to compare the measures of the angles for all given situations (Figure 6, Figure 7, Figure 8, Figure 9).

When the responses of the students to compare the measurements of two congruent angles with different edge lengths were examined, seven students (S5, S6, S16, S18, S21, S22, S25) could not notice that the given angles were congruent. Two of these students (S16, S25), unlike the others, claimed that the angle with a shorter edge is larger than the one with the long edges. As an example of this situation, S25's response is given in Figure 6.

Figure 6.

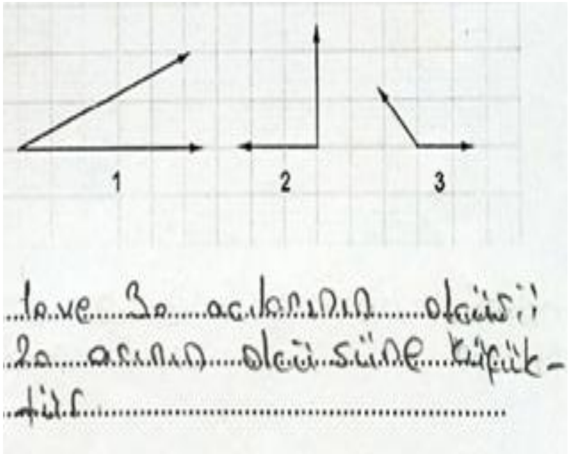
S25's Response



When the responses of the students to compare the measures of the three angles with different edge lengths and the smaller ones with the longest edges were examined, it was seen that two students (S5, S22) made an incorrect comparison. As an example of this situation, S22's response is given (Figure 7).

Figure 7.

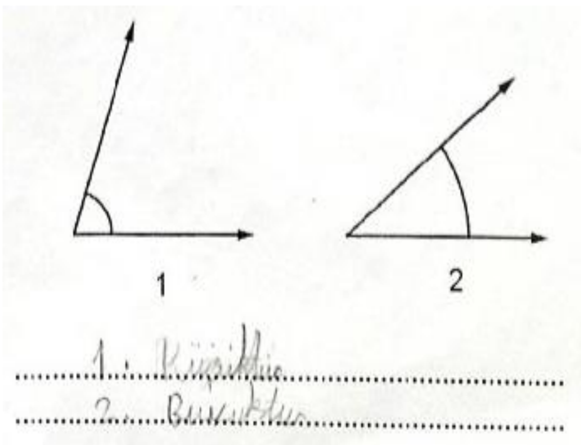
S22's Response



When the findings compare the measure of two angles in the case where the length of the arc that indicates the measure of the smaller angle, or in other words, the radius of the arc that expresses the measure, are longer, it was seen that one student (S5) could not make a correct comparison.

Figure 8.

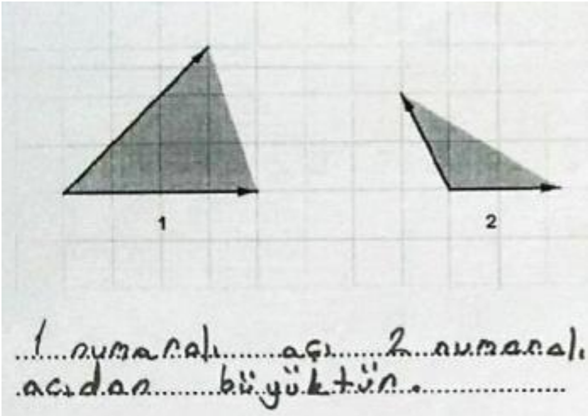
S5's Response



When the findings compare the measure of two angles in the case where the smaller one has a more shaded area in the interior region than the larger one,it was observed that three students (S5, S10, S22) could not make a correct comparison. S10's response is given as an example for this situation (Figure 9).

Figure 9.

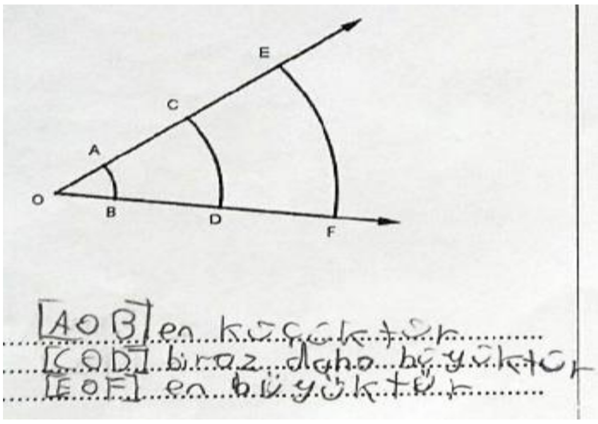
S10's Response



In another question, students were asked to compare angles, which defined as different but locatedin the same angle as presented in Figure 10. When the findings were examined, it was found that one student (S15) could not answer the question, and nine students (S1, S4, S5, S8, S10, S13, S18, S20, S22) could not see that there was only one angle given in the figure and expressed that the measure was larger than the others in the case of the longest arc. As an example of this situation, S1's answer is given in Figure 10.

Figure 10.

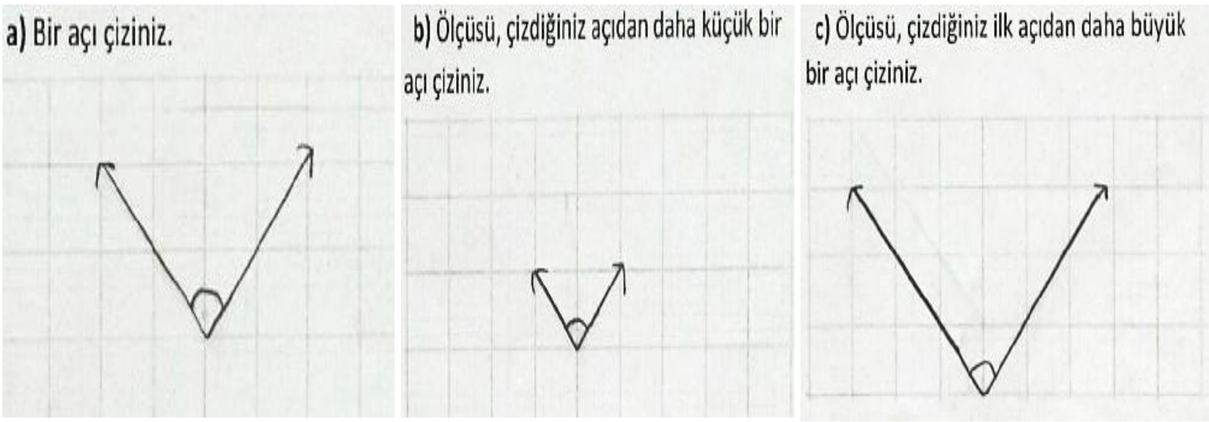
S1's Response



In another question related to this theme, students were first asked to draw an angle. Then, they were asked to draw another angle with measures smaller and larger than the one drawn (Figure 11). When the findings were examined, it was observed that six students (S5, S13, S15, S21, S22, S25) could not give the expected answer. Three of these students (S5, S13, S22) drew an angle by drawing the edges of the first angle as short and long. For example, the angles drawn by S22 are given in Figure 11. It is a remarkable finding regarding students' misconception that S5 and S22 made the same error in the questions given in Figure 6 and Figure 10.

Figure 11.

S22's Response



Findings regarding Adjacent, Complementary, Supplementary and Opposite Angles

Table 6 shows the difficulties and misconceptions about adjacent, complementary, supplementary, and opposite angles.

Table 6.

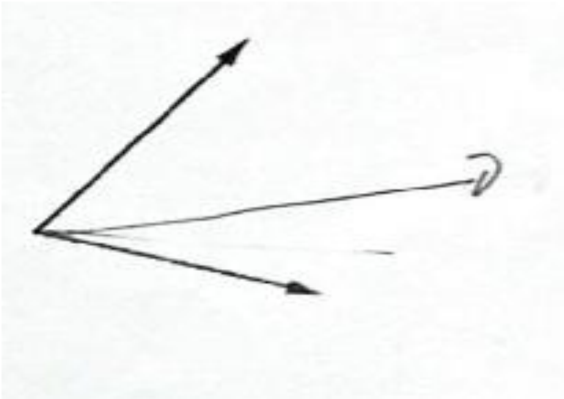
Difficulties and Misconceptions about Adjacent, Complementary, Supplementary and Opposite Angles

Codes	Students	f
Draw an angle adjacent to an angle	S1, S8, S18, S21, S22	5
Cannot define complementary angle	S1, S3, S4, S5, S8, S13, S14, S16, S22, S23, S24, S25	12
Cannot draw complementary angle	S3, S4, S5, S8, S13, S18, S21, S24, S25	9
Cannot define supplementary angle	S1, S3, S4, S5, S8, S13, S14, S16, S22, S23, S24, S25	12
Cannot draw supplementary angle	S3, S4, S5, S8, S18, Ö20, S21, S22	8

When the findings on drawing adjacent angles were examined, it was seen that five of the students (S1, S8, S18, S21, S22) could not draw an angle adjacent to an angle, and three of them (S1, S8, S21), instead of drawing an angle adjacent to the given angle, as seen in Figure 12 they drew a ray in the interior region of the given angle and split the angle into two adjacent angles.

Figure 12.

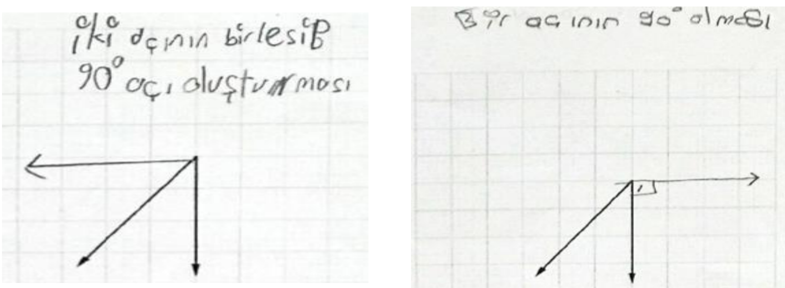
S21's Response



When the findings for complementary angles are examined, the definition knowledge about this concept is discussed. Here, it was seen that 13 students could make a correct definition in general, seven students (S1, S4, S5, S8, S13, S24, S25) could not make the correct definition, and five students (S3, TS4, S16, S22, S23) could not make any explanation. After defining the complementary angle, the findings related to its drawing were examined. When the students were asked to draw the adjacent complementary pair of the given angle, 16 students were able to draw the adjacent complementary pair of the given angle (Figure 13.a), while nine students (S3, S4, S5, S8, S13, S18, S21, S24, S25) could not draw (Figure 13.b). However, although two students (S18, S21) could make a correct definition, they could not draw the angle which is the adjacent complementary pair of the given angle. In addition, a student (S1) who could not make a correct definition and four students (S14, S16, S22, S23) who could not define the complementary angle were able to draw the expected drawing.

Figure 13.

Students' Responses about Complementary Angle



a. S9's response **b.** S25's response

When the findings onsupplementary angles are examined, the definition of this concept is discussed first, similar to complementary angles. When students were asked to define supplementary angles, 13 students were able to make a correct definition in general, six students (S1, S4, S5, S8, S13, S25) could not make the correct definition, and six students (S3, S14, S16, S22, S23, S24) could not explain. After defining the supplementary angle, the findings related to its drawing were examined. When the students were asked to draw the adjacent supplementary pair of the given angle, it was seen that 17 students could draw the adjacent supplementary pair of the given angle, while eight students (S3, S4, S5, S8, S18, S20, S21, S22) could not draw it. When the drawings made by the students were evaluated together with the findings for the definition, three students (S18, S20, S21) could give a proper defitinion but could not draw the angle which is the adjacent supplementary pair of the given angle (Figure 14.a). However, it was observed that three students (S1, S13, S25) who could not make a correct definition and four students who did not make any explanation (S14, S16, S23, S24) were able to draw as expected. In Figure 14.b, it is seen that S25 can draw supplementary angles even though s/he could not make a correct definition.

Figure 14.

Responses of S21 and S25

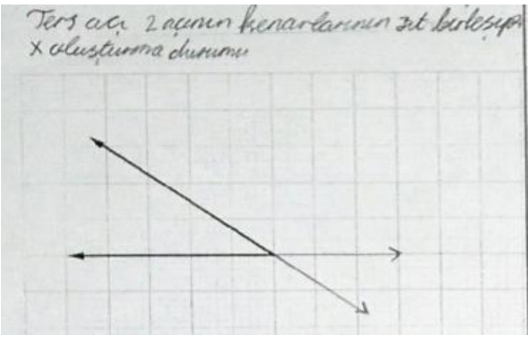
a. S21's response **b.** S25's response

When the findings on the definition of opposite angles were examined, it was seen that only two students (S6, S19) could define opposite angles correctly. It was observed that 14 students could not make a correct definition, and 9 students (S3, S9, S11, S13, S14, S20, S22, S23, S25) could not provide any explanation.

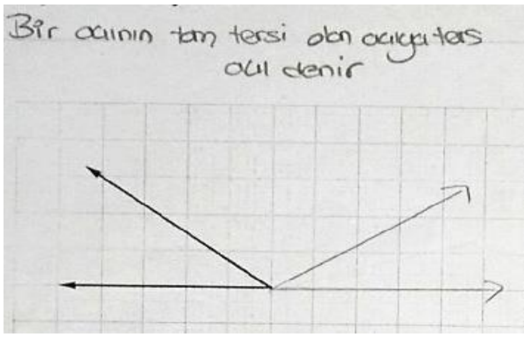
The students were asked to draw an opposite angle of the given angle after defining opposite angles. When these drawings were examined, it was seen that 11 of them were able to draw as expected. However, when these drawings are examined together with their definitions, there were two students (S6, S19) whose description and drawing are sufficient, five students (S1, S2, S15, S16, S21) who could not make a correct definition, and four students (S9, S11, S14, S22) who did not make any explanation about the definition were able to draw the desired drawing. In addition, most of the students who made incorrect drawings drew only one side of the angles as opposite rays (S3, S4, S5, S7, S8, S13, S18, S20, S24). Figure 15 shows examples of definitions and drawings made by students for opposite angles.

Figure 15.

Definitions and Drawings Made by Students for Opposite Angles



a. S19's response



b. S24's response

Constructing A Perpendicular to Any Line from A Point on The Line or Not on The Line

Finally, the students' knowledge and errors about drawing a perpendicular from a point outside and on a given line were examined. The difficulties and misconceptions of students regarding this theme are shown in Table 7.

Table 7.

Difficulties and Misconceptions Regarding Drawing a Perpendicular to a Line from a Point on or Not on a Line

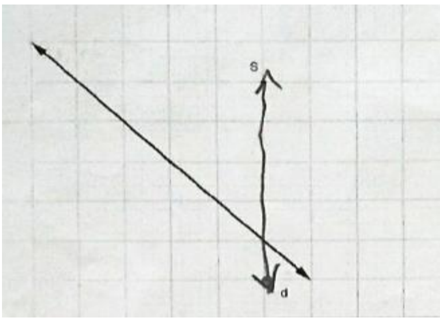
Codes	Students	f
Drawing a perpendicular to a line from a point on or not on the line	S2, S5, S22	3

When the findings on drawing a perpendicular to a line from a point not on the line were examined, it was seen that except for three students (S2, S5, S22), all students were able to draw correctly. In the findings of the students who could not draw correctly, it was observed that there was a tendency to position the perpendicular parallel to the side of the paper (Figure 16.a).

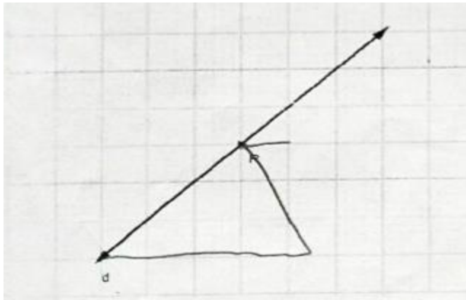
When the findings regarding drawing a perpendicular to a line from a point on the line were examined, it was seen that two students (S5, S22) could not draw the perpendicular, and one student (T8) did not draw anything (Figure 16.b).

Figure 16.

Responses of S2 and S5



a. S2's response



b. S5's response

Results and Discussion

The research concludes that the students have knowledge gaps and misconceptions about the definition of an angle, its drawing and representation with symbols, specifying its vertex and edges, its measure, types, and drawing perpendicular to a line.

The most apparent misconception of the students regarding the definition of an angle is that they state the angle as "the wideness between two rays with the common endpoint", while the other student explanations consist of expressions related to the types of angles, describing the angle and expressions using an example. Here, it can be said that students have similar difficulties when their experiences with the definition and measure of angle are examined as in the historical process (Keiser, 2004). As Keiser (2004) stated in his study, students making mistakes by using more than one expression about the concept of angle seems like a normal result of the multifaceted nature of this concept as in the historical development process. Erbay (2016), on the other hand, stated that the gaps in the angle definitions of the students might stem from the lack of comprehensive concept learning at school, rather than learning a different or incorrect definition.

Studies (Cetin & Dane, 2004; Kilic, Temel & Senol, 2015; Yazgan, Argun & Emre, 2009; Yeşildere, 2007) show that teachers also have misconceptions about the definition of angle. Kilic, Temel, and Senol (2015) also identified in their study that primary school teacher candidates and teacher candidates who receive formation training in the field of mathematics teaching also have this misconception about the concept of angle. This situation makes us think that this misconception of the teachers documented in these studies may affect students' perceptions about the concept of angle. In the studies of Cetin & Dane (2004), it was determined that primary school teacher candidates could not define and apply the essential concepts in geometry. Considering that students encountered the concept of angle for the first time in the first stage of primary education, it can be thought that classroom teachers' perceptions can influence their teaching and therefore explain the difficulties and misconceptions that students have. In this study, when the students were asked to draw an angle, it was seen that they mostly chose to draw the acute angle and right angle. The tendency to draw right angles as a prototype in angle drawings has been emphasized in the literature (Baldy et al., 2005; Butuner & Filiz, 2017; Devichi & Munier, 2013; Doyuran, 2014). For example, in the study by Baldy et al. (2005), when students were asked to draw any angle, they generally drew right angles and tended not to accept obtuse angle as an angle. The reason why students particularly prefer to draw right angles is explained as their familiarity with quadrilaterals with right angles in the teaching process starting from the first stage of their primary education (Devichi & Munier, 2013). The fact that the students' learning processes in the first stage of primary education could not be followed in this research suggests that their tendency to draw the right angle as a prototype may be due to their prior learnings.

When students asked to represent an angle with a symbol, it was also determined that most of the students had difficulties using the correct symbols, and even though they could identify the corner correctly, they had errors in the representation. Some students made errors like using an angle symbol while representing vertex or edges of an angle and failed to state that the vertex of an angle is a point. However, when the students were asked to draw a certain angle, it was seen that the students did not have any trouble in drawing the angle given with the symbol. Doyuran (2014) also reported similar student errors in the angle representation with symbols, as seen in this study. These errors made by the students in using symbols could be due to insufficient emphasis on the use of symbols in the teaching process. Since mathematics is explained with the help of symbols, it should help students learn and use this symbolic language (Calikoglu Bali, 2002).

It is seen that the most common error made by the students in specifying the vertex and edges of an angle is the tendency to "express the vertex and edges of an angle as angles". In addition, many students did not emphasize that the vertex of the angle is a point, and a few students made errors such as drawing the angle incorrectly or drawing the arms of the angle as a straight line. It is thought that this may be due to the inability to provide students with meaningful learning about the concept of angle edge due to reasons such as not being able to comprehend the components of the angle in the teaching process or not being able to connect the point-ray-angle concepts sufficiently. In this case, to overcome the students' difficulties and misconceptions, it may be necessary to review the teaching process and teach for addressing these encountered difficulties.

It is seen that the students have the misconception that "the angle measure changes depending on the arm length". When the literature was examined, similar results were found about the angles (Devichi & Munier, 2013; Keiser, 2004; Mitchelmore, 1998). Doyuran (2014) stated that the reason for this misconception was that students did not know that the arms of the angle were rays. Examining from another perspective, Devichi and Munier (2013) emphasized that students who have a higher tendency to draw prototypes for right angles have a misconception that the measure of the angle changes depending on their arm lengths, but that students who do not tend to draw these prototype drawings develop the concept of wideness while distinguishing different angles in the process of learning the concept of angle.

As to the angle measure, some students had the misconception that "the length of the arc indicating the measure". In other words, they assumed it was "the radius of the arc that indicates the measure". On the other hand, a small number of students had the misconception that "the measure of the angle changes according to the size of the shaded interior region". In the studies conducted, similar results were found that the angle measure changes according to the length of the arc indicating the measure of the angle (Butuner & Filiz, 2017; Keiser, 2004; Mitchelmore, 1998) or that the measure of the angle changes according to the size of the shaded interior region (Butuner & Filiz, 2017). Especially in Keiser's (2004) study, students used various explanations for the angle measure. These included the linear distance between the rays, the ray length, the area between the rays, or the length of the arc drawn between the rays as the angle measure. The existence of several students' difficulties and misconceptions regarding the measure of an angle may be due to the insufficient content knowledge of the teachers, which is one of the most important components of the teaching and learning process. Yazgan, Argun, and Emre (2009) concluded that mathematics teachers do not have sufficient knowledge about the concept of "angle" and "measure of an angle" in terms of the presentations they use. It was observed that teachers had insufficient content knowledge on the concept of angle. It was stated that this was due to their previous (primary, middle and secondary) education and that they could not close the gap between what they learned in the undergraduate curriculum and what they taught to students in schools. In this context, Yesildere (2007) also stated that 20% of primary school pre-service mathematics teachers used mathematical concepts incorrectly, and one of the errors encountered was the use of the concept of "angle" instead of "angle measure". In this context, teachers need to develop their content knowledge by providing pre-service and in-service training to plan the teaching process by identifying possible misconceptions that their students may have.

This study concluded that the students had various difficulties in defining and drawing adjacent, complementary, supplementary, and opposite angles. In his study, Erbay (2016) mentioned that sixth-grade students' knowledge of adjacent, complements and supplementary angles is limited based on rote-memorization and that they are unsuccessful when faced with different questions than they are accustomed to. In their study, Taylan and Aydin (2018) emphasized that sixth-grade students could not define these angles even though they did the calculation questions about these angles and explained the reason for this situation by the widespread use of multiple-choice exams in our country. In this study, it was seen that the students experienced various difficulties because they were worked on questions that were not based on any calculations and were different from the ones they were accustomed to. In the teaching process of geometry subjects, question types should be diversified in defining and drawing adjacent, complementary, supplementary, and opposite angles, and questions for conceptual understanding rather than procedural understanding should be included in this process.

Some students' knowledge gaps and errors regarding "drawing a perpendicular to a line from a point outside or on the line" were determined. Butuner and Filiz (2017) mentioned a similar misconception in their study and especially mentioned that students believe that lines drawn parallel to the long side of the paper are perpendicular to any given line and that a perpendicular line cannot be drawn from a point on a line. However, Karakus (2014) emphasized that constructing a perpendicular line from a point outside or on a line is stated by Smart (1993) as one of the basic constructions in Euclidean geometry and that it is effective in solving more complex geometry problems. In this context, as seen in this study, the teaching process of drawing perpendicular to a line, which is one of the geometric construction topics that students have difficulty with, should be enriched and opportunities should be provided for students to make drawings on this subject, which is often not emphasized.

As a result, this study identified many difficulties and misconceptions of students about angles. Studies (Baldy et al., 2005) also confirm that students have difficulties verbalising an angle's properties, drawing an angle, and clearly defining angles. Teachers and teacher candidates should consider these difficulties and misconceptions of students regarding concepts and orchestrate their learning environments accordingly. However, studies reveal the inadequacies of teacher candidates and teachers. This study and similar studies will support teachers to reveal these situations.

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Authors	*Contact*
Deniz OZEN UNAL, mathematics education, teaching geometry, teacher professional development, technology in mathematics education.	Aydın Adnan Menderes University, Faculty of Education, Department of Mathematics Education, Efeler, Aydın, Turkey. E-mail: deniz.ozen@adu.edu.tr
Omer URUN, mathematics education, gifted education, college mathematics.	Nazilli Science and Art Center (BILSEM), Nazilli, Aydın, Turkey. E-mail: omerurun@gmail.com

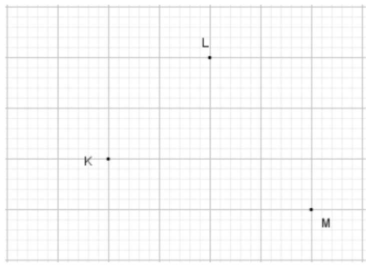
Appendix 1. Sample open ended problems

Definition of an angle

1. Açının tanımını veriniz. Bir açı çizerek isimlendiriniz ve sembolle gösteriniz.

Drawing angle

3. Aşağıdaki verilen K, L ve M noktalarını kullanarak $\widehat{KLM}$ nı çiziniz.



Representation of Angle

4. Kenarları [DE ve [DF olan açı aşağıdaki sembollerden hangileri ile gösterilebilir?

$\widehat{D}$ ....☐

$\widehat{E}$ ....☐

$\widehat{F}$ ....☐

$\widehat{EDF}$ ....☐

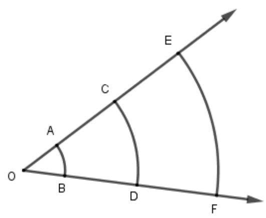
$\widehat{FDE}$ ....☐

$\widehat{DEF}$ ....☐

$\widehat{DFE}$ ....☐

Angle Measurement

8. Şekildeki $\widehat{AOB}$, $\widehat{COD}$ ve $\widehat{EOF}$ açılarının ölçülerini karşılaştırınız.



....

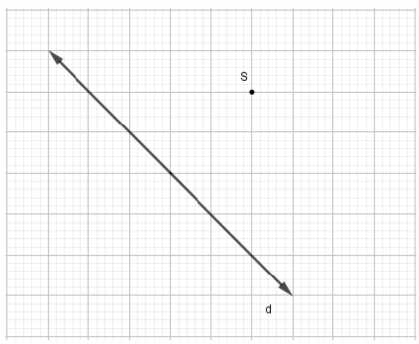
....

....

Adjacent, complementary, supplementary, and opposite angles.

Constructing a perpendicular to any line from a point on the line or not on the line

15. Aşağıda verilen d-doğrusuna S noktasından dikme çiziniz.



10. Tümler açılar nedir? Açıklayınız ve aşağıda verilen açının komşu tümlerini çiziniz.